Soft Tissue Deformation Simulation in Virtual Surgery using Nonlinear Finite Element Method

Zhennan Yan, Lixu Gu*, Pengfei Huang, Sizhe Lv, Xiao Yu and Xianming Kong*

Abstract—**Simulation for soft tissue's realistic deformation is an important part in Virtual Surgery. For large global deformation of soft tissue, linear elastic models are inappropriate, such as Mass-Spring and linear Finite Element Method (FEM). In this paper we present a simulation for 3D soft tissue using nonlinear strain computation. To get a finer mesh for FEM, we consider meshing algorithm based on Improved Delaunay criterion. Besides, we would present Spatial Hashing Collision Detection method and some improvement for real-time computation.**

I. Introduction

Virtual surgery is a promising application to train intern surgeons. It aims at reducing the costs and risks in surgical training. In its implementation, it displays the model of human tissue on the computer monitor and simulates the motions and deformations of the tissue. It also provides interfaces through which the intern can use common surgery tools to manipulate the artificial soft tissue.

Although the simulation of human soft tissue has a long history in biomedical engineering and computer science, physically realistic modeling and computation of soft tissue's deformation has been the bottleneck of many applications, especially virtual surgery system. So far many simulations of deformation have been implemented using simple models: Mass-Spring [1], linear elastic FEM [3], [4], centerline [11], MM-Model [12] and so on. These methods work well for simulations of very small strains and local deformations, but have poor accuracy for large global deformation modeling.

In previous researches, the speed and robustness of the models are the main interest for surgery simulation systems, but the accuracy is of less concern. Nowadays, realistic simulation becomes more important in some virtual surgeries. In facial plastic surgery simulations, for example, the doctors and the patients are concerned about what it will look like after surgery, while they need a realistic simulation with high accuracy.

Global deformation [2] is commonly happened in medical domain and worth simulating for virtual surgery, such as large twisting or bending of an object, which involves the entire body. For gaining accuracy besides visually satisfactory rendering, we apply nonlinear Finite Element Method to model large global deformations.

Many problems in mechanics and physics lead to differential equations, and most of which are impossible to be solved in analytic way. FEM ([2]-[7]) is such a numerical method that subdivides the object to a finite set of primitives, such as tetrahedral mesh, with a physical equilibrium equation for each of them. With application of variational principle, it can transform to problems of solving large systems of linear equations. And Conjugate Gradient algorithm [8] is a popular algorithm for solving large sparse symmetrical linear systems.

In this paper, first, we would discuss the meshing method, and then focus on simulation for large deformation of soft tissue in virtual surgery using nonlinear FEM in contrast with linear FEM. Besides, we would consider collision detection [10], as well as the methods for reducing complexity and expense of computation. At last, we would illustrate our experimental observations based on 3D kidney model.

II. Improved Delaunay Meshing

To simulate a continuum solid object using computer, we must discrete it into a number of elements firstly. In this paper, we generate tetrahedral mesh based on Delaunay criterion.

By far most of the tetrahedral meshing techniques are utilizing the Delaunay criterion. The criterion states that for n-dimensional cases a circum-sphere of each simplex within the mesh contains only the n+1 defining points of the simplex.

There are some open source tools that can create mesh for 3D objects maintaining Delaunay criterion, such as the Visualization ToolKit (VTK). But the VTK can't preserve initial boundary, and produces sliver tetrahedrons if the surface is complex.

We have improved the Delaunay algorithm to generate a well-proportioned, boundary preserved mesh for latter deformation. The following is the steps we should take: 1) find the circum-cube of the bound box which contains the source object represented by boundary points and facets, and mesh the cube into 5 tetrahedrons; 2) subdivide the circum-cube into small cubes and store all vertices of the cubes; 3) disarrange all points gotten in step 2 within a small range to decrease the probability of 4 points co-planarity; 4) generate interior points from output of step 3; 5) insert boundary points and interior points into the 5 tetrahedrons gained in step 1 according to the Delaunay criterion; then, check boundary points and topology preservation; last, deal

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with sliver tetrahedrons. We detect the sliver tetrahedrons by calculating the standard deviation of their edges and comparing with a defined threshold. If the standard deviation is bigger than threshold we would re-mesh the sliver tetrahedron with its neighbors until satisfying.

This meshing technique is of a little high time complexity, but it generates more regular and smooth mesh in contrast with traditional Delaunay method. It can be seen from comparison between two pictures of fig.1.

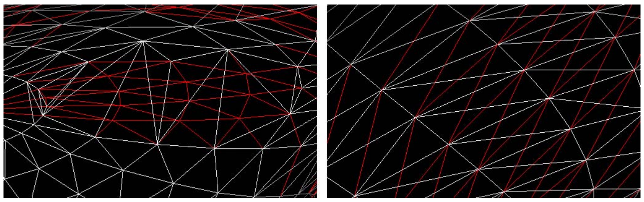

Fig. 1. Left picture shows a mesh generated by traditional Delaunay 3D Meshing. Right picture shows a mesh generated by the Improved Delaunay Meshing.

III. NONLINEAR DEFORMATION

The theory of elasticity is a fundamental discipline in studying continuum materials. It consists of equilibrium equations, kinematics equations, constitutive equations and boundary conditions. Synthesizing all those equations allows us to establish a relationship between the deformation of object and external force. But in most cases, an analytic expression of this relationship is impossible. Finite element method is one way to solve such problems.

FEM is one of the most popular and stable numerical methods in engineering analysis. In this paper, we only discuss elasticity in the context of FEM.

Using this method, we should follow these steps: discrete the volumetric solid into a number of finite elements, which has been discussed in previous section; select the displacement function, and we use linear interpolation in tetrahedral mesh; construct element stiffness matrices considering physical equilibrium; assemble element matrices into a global stiffness matrix; solve the system of equations; calculate other unknowns such as strains and stresses. In implementation, we only calculate displacements at vertices of each tetrahedron, the values at other points within the elements are interpolated by a weighed sum of all the nodal displacements.

A. Static Deformation

For static deformation, it is required to solve the following system of equations

$$R(a) = F \tag{1}$$

where $a = [u_1 \quad v_1 \quad w_1 \quad \cdots \quad u_n \quad v_n \quad w_n]^T$ is the $3n$-dimensional nodal displacement vector for 3D objects, and u, v, w are the corresponding displacement variables at given point; $R(a)$, the internal force vector due to deformation; F, the external force vector.

Usually the engineering strain vector is defined as

$$\varepsilon = [\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{zx}]^T. \tag{2}$$

Many previous works ([3], [4]) using linear strain as following:

$$\varepsilon_x = \frac{\partial u}{\partial x} \tag{3}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{4}$$

where x, y, z are the independent variables of the Cartesian frame. Other terms of the strain are defined similarly.

This linear strain vector makes the internal force vector $R(a)$ linear to the nodal displacement vector a. Namely equation (1) can be written as following linear system:

$$Ka = F \tag{5}$$

where K is the constant stiffness matrix independent with a.

However, the approach is only suitable for simulating small local deformations, e.g., poking and small bending. Using the linear strain to simulate large global deformations will cause distortion. It is because that this linear strain models rigid motions as differential motions. If we subject a large rigid body bending to an undeformed object, the linear strain (3) and (4) will give a non-zero strain, but the object should not have any deformation actually.

In virtual surgery, large global deformations are crucial. To simulate global deformations, we use quadratic strain in stead. So the strain vector become following expressions:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left[(\frac{\partial u}{\partial x})^2 + (\frac{\partial v}{\partial x})^2 + (\frac{\partial w}{\partial x})^2 \right] \tag{6}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left[\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \right] \tag{7}$$

This nonlinear strain vector makes the internal force vector $R(a)$ dependent to a. So equation (1) now becomes a nonlinear system. We can rewrite $R(a)$ as the following expression:

$$R(a) = (K + P(a)) \cdot a \tag{8}$$

where K is the same as the constant matrix derived from linear strain, $P(a)$ is the term depends on the nodal displacement vector a.

To solve this nonlinear system of equations, we use Newton-Raphson method as following:

$$a^{(n+1)} = a^{(n)} + \Delta a^{(n)} \tag{9}$$

$$\Delta a^{(n)} = (K + P(a^{(n)}))^{-1} \cdot (F - R(a^{(n)})) \tag{10}$$

This is an iterative numerical solution, and the main cost is calculating the inverse of matrix. Because the stiffness matrix is large, sparse and symmetric, we use Conjugate Gradient [9] method to solve the system of equations.

The simulations of static linear versus nonlinear strain deformation for an elastic beam are shown in fig.2. In the picture, the green mesh represents the original state, while the yellow and red ones as the state caused by body force, for example gravitation. We can see that the yellow mesh is stretched under the body force, which is distortion, while the red one simulates the deformation realistically.

Soft tissue is much more complex, and usually there are many large motions in surgery. So, if we want to get realistic simulations for soft tissue's deformation, we should apply nonlinear strain models.

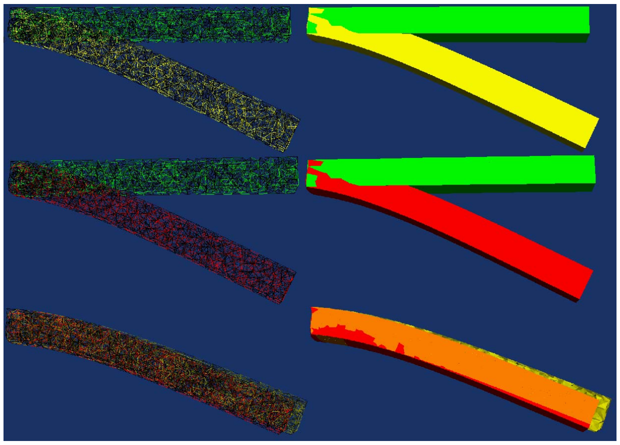

Fig. 2. Simulations for static linear strain deformation and nonlinear strain deformation are shown respectively. The yellow mesh is linear strain deformation while the red one is nonlinear strain deformation.

B. Dynamical Deformation

To model dynamical deformation of elastic objects, we should solve the following system of differential equations:

$$M\ddot{a} + D\dot{a} + R(a) = F \tag{11}$$

where $\ddot{a}$, $\dot{a}$ are the respective acceleration and velocity vectors; *M*, the mass matrix, *D*, the damping matrix. Actually, equation (11) can become equation (1) leave out of account the acceleration and velocity vectors.

We have to solve the system of differential equations (11) approximately by numerically integrating along time dimension. Namely we have to solve the system of equations at a time step according to the previous time steps.

In this paper, we use center difference method, which is simplicity of Newmark recurrence scheme. It's a single-step explicit integration method, namely it only use values at $t_{(n)}$ to solve the system at $t_{(n+1)} = t_{(n)} + \Delta t$. So we use the following iterative expression:

$$\begin{aligned}&(\frac{1}{\Delta t^2}M + \frac{1}{2\Delta t}D)a_{t+\Delta t} = \\ &F_t - (R(a_t) - \frac{2}{\Delta t^2}M)a_t - (\frac{1}{\Delta t^2}M - \frac{1}{2\Delta t}D)a_{t-\Delta t}\end{aligned} \tag{12}$$

We should solve the system at each time step to get some displacements for dynamical simulation. But fortunately, the mass matrix *M* and the damping matrix *D* usually are constant, and we can approximate *M* by a diagonal matrix [5], then apply Rayleigh damping $D=\lambda M$. These simplify the computation very much. We only need to recalculate the external force *F* and internal force *R(a)* at each step. For soft tissue, the time steps can be large. Besides, we need to initialize $a_0, \dot{a}_0, \ddot{a}_0$ and calculate the following value:

$$a_{-\Delta t} = a_0 - \Delta t \cdot \dot{a}_0 + \frac{\Delta t^2}{2}\ddot{a}_0 \tag{13}$$

It's necessary to point out that the time step Δt must not be larger than a critical time step which has relationship with the smallest element in the mesh [5]. So we want a nice mesh without too small elements or sliver tetrahedrons for soft tissue, that's what we have discussed in section 2.

IV. Collision Detection

Collision detection is very important in any realistic interactive environment, so as in virtual surgery. Generally, collision detection algorithms need to deal with broad-phase which determines all potential collision pairs and narrow-phase which tests intersection between two complex geometrical models.

However, it is challenging when regarding the deformable models. Here we consider an adaptive multi-resolution spatial partitioning algorithm [10] to detect collision occurring between a simple rigid object and a complex deformable body. It divides the space into a number of grid-cells and uses a hash table to store the position information of the deformable objects and their vertexes. We should find an optimal cell-size for each object and map each cell and vertex to a unique address in hash table. A 2D example is shown in fig. 3.

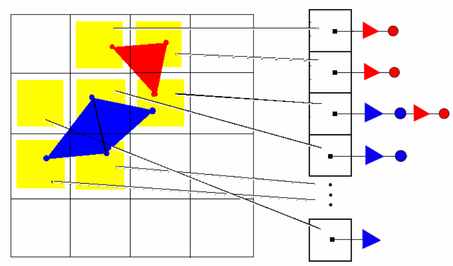

Fig. 3. Map cells occupied by triangles and the vertexes to Hash table. Vertexes and triangles in the same Hash table index should be tested for intersection.

In simulation, the grid-cells each object occupies are computed at each time step and all occupied cells have references to the corresponding objects. Then, for each vertex in the scene, test it for intersection with other objects that occupy the same grid-cell.

V. Graded Mesh

The computational cost for 3D deformation is much higher than 2D mesh using FEM, because the number of elements increases rapidly. A way to decrease the size of system generated using FEM is to apply Graded Mesh [2]. A 2D example of triangular mesh is shown in fig. 4. Such a graded mesh can be extended to 3D tetrahedral mesh, and it can reduce the complexity of 3D problems from $O(n^3)$ to $O(n^2)$, while losing little accuracy. The algorithms discussed in Hebert's ([9]) symbolic local refinement of tetrahedral mesh can be directly applied to generate a graded tetrahedral mesh in 3D.

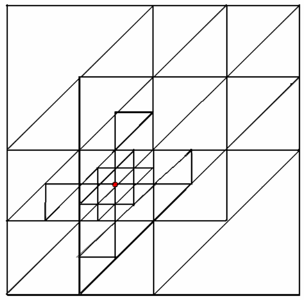

Fig. 4. 2D triangular Graded Mesh.

VI. EXPERIMENTS

We generate a human kidney mesh using the meshing technique discussed in section 2 and simulate its deformation under a boundary force. Fig. 5 shows the results. And observations are showed in table 1.

We can see in fig. 5 and table 1 that nonlinear strain deformation is closer to the realistic situation, while the linear strain deformation is more distorted.

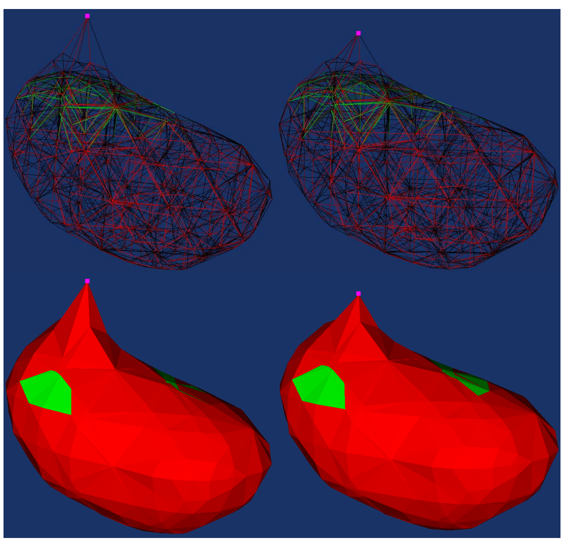

Fig. 5. Left is linear strain deformation of human kidney, while right is nonlinear strain deformation. Green meshes represent the original state, while the red ones represent deformation state that is caused by an equal external force at the purple point.

TABLE I

Item	Mesh state	Quantity
Volume	Original state	332889 (mm^3)
	Linear strain deformation	364543 (mm^3)
	Nonlinear strain deformation	352602 (mm^3)
Surface Area	Original state	208017 (mm^2)
	Linear strain deformation	211821 (mm^2)
	Nonlinear strain deformation	210744 (mm^2)

However, solving the system of equations takes most percent of total time, especially for nonlinear strain deformation, which takes about ten times more than linear situation. In our system, we use Conjugate Gradient algorithm to solve the system, which is suitable to implement onto the GPU [6].

VII. CONCLUSION

In this paper we focus on the simulation for realistic deformation of soft tissue using nonlinear strain FEM, because it's important to simulate large global deformation in virtual surgery. And through experiments, we take the advantage of using nonlinear strain deformation to simulate soft tissue. To achieve real-time performance, we apply numerical technology, graded mesh method and some other techniques.

As a future work, the GPU acceleration for computation will be considered more not only for efficiency but also accuracy. Some other technologies will be also implemented, such as condensation [7] for decreasing the size of system in FEM.

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