# Particle-Based Deformable Modeling with Pre-computed Surface Data in Real-Time Surgical Simulation

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**Abstract.** Particle-based method has proven to be a powerful tool in real-time surgical simulation for its simplicity and high efficiency. However, it is difficult to model the high-resolution surface deformation details with standard particle-based techniques. In this paper, we propose a novel approach to model the elastic behaviors of organs with complex surfaces in surgical environment. The basic idea of our approach is to introduce an auxiliary surface mesh into the existing particle-based simulation framework, and utilize the pre-computed surface data with experimental biomechanics parameters for deformable modeling. The high-resolution organ deformations and the low-resolution soft tissue deformations are treated in surface-based and particle-based methods respectively. Our method provides an efficient and physical valid way to model the organ deformation details for particle-based surgery simulation techniques without using adaptive particle methods, as shown in our experiment results.

**Keywords:** interactive surgical simulation, minimally invasive surgery, deformable modeling, point-based simulation, physically-based modeling.

# 1 Introduction

Interactive surgery simulation techniques play an important role in today's medical training program. Modeling the deformable organs and tissues accurately in surgery scenes is a hot topic in real-time medical simulation. A variety of models has been proposed over the past decades to provide deformation results balancing accuracy and efficiency in real-time virtual surgery simulation (e.g. mass-spring model [13][14], finite element model [6], boundary element model [4][2] or particle-based model [5][7][9][10]).

Different data representations, such as volume mesh, surface mesh or particle systems, are used in different physics models. Because there exists hardly any direct correlation between these structures, to couple these models in one system with high performance is a challenge to real-time surgical simulations. Particle-based simulation methods have been proposed over the past decades to alleviate the simulation problems brought by meshes in the area of computation physics and computer graphics. For its simplicity in topology and data representation, particle-based methods are suitable for simulating the elastic and plastic bodies in real-time applications. And they have been introduced into minimally invasive surgery simulation in [8] and [10].

However, there also remain several problems in pure particle-based methods. One of them is the simulation resolution. Since the objects in the scene are all represented by discrete points with the same support field, it is difficult to describe some deformation details on the organ surface, which is visually important in surgical training. Although some adaptive methods have been proposed in particle-based simulation in [1], they are less efficient in real-time simulation since extra data structures (e.g. octree) are required to handle the points with different resolutions. To model the deformation details without sacrificing the high efficiency, in this paper we propose an alternative method for the adaptive particle approach. In our approach, a surface mesh is re-introduced into the particle-based simulation framework and serves as a high-resolution elasticity solver with pre-computed surface deformation data. Unlike the previous mesh-based methods, the surface mesh in our methods is only used for physics computation, and all the other aspects of simulation, including collision detection, contact handling and haptic rendering, are treated in a particle-based way with the same resolution. A physically-based simulation framework with combined particle-based and surface-based physics representations is applied in simulating laparoscopic surgery training procedures. Insignificant elements in the surgical training scenes, such as small soft tissues and fats, are modeled with a standard particle-based method, while the important target pathologic organs are modeled with our new deformable model to provide high-resolution visual feedback to the trainees.

# 2 Overview

Our method is encouraged by the previous works of particle-based surgical simulation [8][9][10]. It contains three main parts outlined below.

1. Point and Mesh Construction from Medical Data. In this phase the medical data is segmented into different parts representing different organs, bones and tissues. A set of physics points is extracted and surface meshes are constructed for highly detailed simulation of pathologic organs and tumors. Distance field information is constructed based on the particles for fast collision handling.

2. Surface Data Pre-computing for Pathologic Organ Modeling. For the important pathologic organs in surgical training, a deformation surface data is pre-computed using the boundary element elastic solver. The biological material properties measured by biomechanics experiments and the surface geometry information are used in the surface data pre-computation.

**3.** Real-time Deformable Modeling with Hybrid Methods. In runtime simulation, the pre-computed surface data is integrated into the particle-based simulation framework and help to guide the high-resolution organ deformation in a physically valid way. The high-resolution pathologic organs and other ordinary elements in the surgery environment are modeled in different ways.

# **3** Physics Point Extraction and Distance Field Construction from Medical Data

In this stage, organs and tissue models in the surgery environment are segmented from the CT image data. The discrete physics points are extracted from each segmented organ.



Fig. 1. In the data extraction stage, the distance field (left and middle) are constructed and put into the unified Eulerian grid for collision handling

For the low resolution deformable models, these points are directly used in simulation. For the high resolution models, an additional surface mesh is used to represent the surface of deformable object. The surface meshes are generated from medical data by using marching cube algorithm, and the particle surface is generated by sampling the coarse triangle mesh.

All the particles, including the particle surface used for high-resolution models and physics particles used for low-resolution models, are mapped onto a global grid for collision handling. Additionally, as illustrated in Fig. 1, we construct a local Eulerian grid for each immobile or rigid object such as bones and surgical instruments in the surgery scene, and pre-compute a distance field in the local Eulerian grid as in [1], [3] and [11]. The distance field is used in spatial-hashing based collision detection [15] for fast collision detection.

#### 4 Particle-Based Fast Modeling for Small Soft Tissues

The target pathologic organs and the small soft tissues in surgical training are modeled in different ways in our framework. With the extracted physics points in the pre-processing step, we use smoothed particle hydrodynamics (SPH) to simulate the unimportant elements such as small soft tissues and fats in the scene. The deformable model is assumed as a Hookean material. The strain  $\varepsilon$  and the stress  $\sigma$  follow the following rules:

$$\sigma = C\varepsilon$$
 (1)

in which C is a rank four tensor. As for isotropic materials, C can be represented by a 6x6 matrix determined by Young's Modulus and Poisson Ratio of elastic materials as in [7]. In each time step, the surface force, body force and elastic force on each particle are summed up and applied in a common ODE solver to model the deformation. As in Fig. 2, different operations can be exerted on the fast point-based model to change the shapes of the object in a low resolution. We employ this pure particle-based method to model the small and unimportant soft tissues. But for the pathologic organs which usually play a central role in surgical training, a model with higher resolution is needed to model the organ deformation details in the interactions with the surgical instruments.



Fig. 2. Low-resolution deformations of the small soft tissues are modeled by using the pure particle-based method in our framework

# 5 Surface-Guided Modeling for Surgical Organs

To model the pathologic organs in the surgery scene precisely, surface data is precomputed before runtime simulation and is used to provide accurate global deformation information for the particles in real-time deformable modeling. The surface data is a global deformation matrix calculated from the geometry and the material properties of the target surgical organ. It contains the deformation data of all the surface elements with higher resolution than the physics particles and can be used to guide the deformation of the particles in the runtime simulation.

The surface data pre-computing is based on the Boundary element method (BEM), but different from the previous BEM work [4][2][12], we only utilize the surface data (the global deformation matrix) pre-computed by BEM to guide the particle deformation, rather than directly computing the deformation with BEM in each timestep. All the computations related to BEM happen in the pre-computation stage instead of real-time loops. So there's no need to solve the boundary integration equations in each timestep as the standard BEM does. This process is replaced by setting external forces from surgical instruments and internal repulse forces from constraint particles into a boundary traction vector and calculating the global deformation by simply multiplying this vector with the pre-computed deformation matrix. Then the deformation computed from the surface data is regarded as a corrected target position for the physics particles. It backwardly corrects the displacement of these discrete particles and interacting with other particles in a Lagrangian way. The hybrid deformable model consists of two main parts: the surface data computation in the pre-computing stage and the surface guided particle deformation in real-time simulation.

#### 5.1 Surface Data Pre-computation

In the surface data pre-computing stage, material parameters for a surface model are determined by biomechanics experiments, and the global deformation matrix of surface elements is initialized by using the geometry information of the coarse surface mesh and the measured biology material parameters. Three material parameters are used in the surface data pre-computing: Young's Modulus E (100-110kPa), Poisson's ratio  $\sigma$  (0.46-0.50) and density(1.00-1.12gcm-3). The former two parameters are used in surface data precomputing and are measured by determining the biomechanics

strain-stress relationships of tissues sampled from different parts of porcine kidney and liver in INSTRON mechanical testing machine as in [10].

With the biological material parameters and the surface geometry shape of the organ, the deformable surface data is calculated based on the Navier's equation of isotropic elastic material. The Green-Gauss theorem and the Kelvin fundamental solutions of linear elastostatic problem is used to set up an integral equation defined on the boundary of domain  $\Omega$  and numerically discretized into a matrix formulation as

$$\mathbf{C}\mathbf{u}_{i} + \sum_{j=1}^{N} \left( \int_{\Gamma_{j}} \mathbf{p}^{*} d\Gamma \right) \mathbf{u}_{j} = \sum_{j=1}^{N} \left( \int_{\Gamma_{j}} \mathbf{u}^{*} d\Gamma \right) \mathbf{p}_{j} \quad (i = 1, \dots, n)$$
(2)

Take the summary of left and right side as H and G. To compute the integral term in  $\mathbf{H}_{ij}$  and  $\mathbf{G}_{ij}$ , Gauss integration method is employed on surface element i and j. Substituting Kelvin fundamental solutions into Gauss integration formula, we get

$$\mathbf{H}_{ij} = \left| S_j \right| \boldsymbol{\omega}_k \sum_{k=1}^{7} \mathbf{u}_{ij}(P, Q_k)$$
(3)

$$\mathbf{G}_{ij} = \left| S_j \right| \boldsymbol{\omega}_k \sum_{k=1}^{7} \mathbf{p}_{ij}(P, Q_k)$$
(4)

where  $S_j$  is the area of element j,  $\omega_k$  are Gauss coefficients and P, Q are sampled points on the triangle surface. In our implementation we use the seven point integration method for each triangle element. Then we get the linear equation system

$$\sum_{j=1}^{N} \mathbf{H}_{ij} \mathbf{u}_{j} = \sum_{j=1}^{N} \mathbf{G}_{ij} \mathbf{p}_{j} \quad (i = 1, \cdots, N)$$
(5)

**H** and **G** are  $3N \times 3N$  matrices, each containing  $N \times N$  sub-matrices  $\mathbf{H}_{ij}$  and  $\mathbf{G}_{ij}$ . The values in each sub-matrix are calculated based on the material parameters and the relative positions of the two triangle elements as in the standard BEM [4][12]. **U** and **P** are  $1 \times 3N$  vectors composed of displacement and traction vectors of N surface elements. Then the global deformation U is computed as

$$\mathbf{U} = \mathbf{H}^{-1}\mathbf{G}\mathbf{P} = \mathbf{M}\mathbf{P} \tag{6}$$

where  $\mathbf{M}$  is the global deformation matrix. Computing  $\mathbf{M}$  is a time exhausting task since it is a non-sparse and asymmetric matrix. But this surface data needs to be computed only once in the preprocessing stage and can be used to guide the surface deformation in runtime simulation.

#### 5.2 Runtime Organ Deformation Simulation

In runtime simulation, the physically accurate global deformations are computed based on the global deformation matrix  $\mathbf{M}$  according to the current boundary condition (by multiplying the matrix  $\mathbf{M}$  with the surface traction vector), and then



**Fig. 3.** Comparison between particle-based deformation (left) and our surface-guiding (right) method. High-resolution deformations are modeled with the pre-computed surface data.

served as target positions for the particles on the surface. As illustrated in Fig. 3(right), in each time step, displacements of physics particles are corrected based on the well-defined goal positions by virtual springs. They are used to describe the organ dynamic temporal deformations such as viscosity and creeping in runtime simulation. The values are determined by matching the continuous images taken from dynamic deforming organs.

The interactions between the deformable model and VR environment, including collisions, contacts and dynamic simulations, are handled by the particle surface using point-based techniques. So there is no interaction between the fine surface elements and the coarse physics particles with different support radius. All the computations are based on physics particles in the point-based framework, while the precomputed matrix **M** is the only extra data used in real-time simulation.

With the precomputed deformation matrix  $\mathbf{M}$ , it is easy to compute the accurate deformation position for each physics particle in real-time without employing any adaptive particle method. As illustrated in Fig. 4, the local deformation of the pathologic liver in response to the surgical operations on it is modeled easily with the pre-computed matrix, which cannot be resolved with the low number of physics points (less than 1,000 in this case).



**Fig. 4.** The surface-based deformable model is integrated into our particle-based surgical minimally invasive surgical simulator. Deformation details of the liver organ in response to the surgical operations are modeled in real-time.

# 6 Experiments and Evaluations

Our particle-based deformable model with pre-computed surface data is integrated into a virtual reality training system for laparoscopic surgery as in Fig. 5. The training system is set up based on particle-based simulation techniques. Surgeons are trained to perform operations on pathologic organs with different biological parameters and acquire specific skills under different conditions. The simulation framework is implemented in C++ and rendered using OpenGL. All the experiments were carried out on a 2.26GHz Pentium M notebook PC with GeForce 9650M graphics card and 2 GB of memory.



Fig. 5. Our virtual reality laparoscopic surgery training system employing the particle-based techniques and our novel model for pathologic organ deformations.

To evaluate the time performance of the surface-guided deformable model, the simulation method is tested on five models with different surface polygon numbers ranging from 600 to 2200. As in Table 1, summing the computation time of surface deformation, position correction and surface interpolation for each model, it can be seen from the overall time cost of each object that our hybrid model is no more than 5 ms and is suitable for real-time applications. Comparing with the standard particle-based method in [10], which can provide 60 fps simulation results, our particle-based method utilizing the pre-computed surface data can work within the same time restrictions (no less than 50 fps).

In the second experiment, we compare the accuracy of our method with the standard BEM. In our hybrid model, the deformation of the surface is computed by

Model	Point	Surface	Position	Surface	Overall
	Number	Computation(ms)	Correction(ms)	Interpolation(ms)	(ms)
1	284	0.63	0.228	0.137	1.00
2	508	1.13	0.309	0.266	1.67
3	784	1.75	0.653	0.422	2.83
4	876	1.94	0.725	0.497	3.162
5	1144	2.53	1.012	0.634	4.176

Table 1. Time performance of the surface-based deformable model

utilizing a pre-computed surface data, and interpolated back to the discrete particles. Compared with traditional surface-method such as BEM, it is unavoidable that some deformation details may be lost during interpolation between surface and particle models. We measure this accuracy loss by comparing our method with the standard BEM method as in Table 2. The accuracy loss ratio R is defined as  $(\sum \|\mathbf{u}_i^o - \mathbf{u}_i^{imp}\|)/(\sum \|\mathbf{u}_i^o\|)$ , in which  $\mathbf{u}_i^o$  is the vertex displacements computed by using standard BEM model, and  $\mathbf{u}_i^{imp}$  is vertex displacements computed using surface interpolation.

Vertex	Vertex	Polygon	Vertex	Accuracy
Number	Number	Number	Num	Loss Ratio
(Coarse)	(Fine)	(Fine)	Ratio	
284	256	508	0.90	2.1%
	394	784	1.39	3.7%
	574	1144	2.02	5.1%
608	506	1008	0.83	1.4%
	1000	1996	1.64	3.3%
	1586	3168	2.61	6.2%

Table 2. Relationship between vertex number ratio and accuracy loss ratio

It is concluded from Table 2 that for most organ models in surgical simulation with relatively smooth surface, this accuracy loss can be restricted within an acceptable scale if the ratio of coarse surface vertex number to fine vertex polygon number is below a threshold. When the vertex number ratio is below 2, the accuracy loss ratio can be restricted below 5%-6%, which is not obvious in visualization. Considering that the time complexity of pre-computing global deformation matrix is  $O(n^2)$ , and updating global matrix in real-time is O(n), it is worthwhile to accelerate computing time 4-5 times in the cost of tiny visualization lost.

Compared with the pure particle-based method, our hybrid model can produce high-resolution local surface details in response to surgical instruments as in Fig. 4, and these details cannot be resolved on the physical particle level. There are also some limitations of our approach. The temporal elastic behavior based on spring parameters is less accurate than those based on continuous mechanics. The main purpose of our approach is to simulate the high-resolution organ surface deformation as accurately as possible (by using precomputed data) and in other aspects to provide trainees visually plausible feedbacks. Future work includes designing more accurate biomechanical experiments to measure the real material parameters and applying the method to more complex and heterogeneous models.

### 7 Conclusion

In this paper we propose a new method to model high-resolution organ deformations within particle-based simulation framework. We utilize the precomputed surface data

with experimental biological material parameters to model the deformation details on the surface of important organs in a physically-based way. Our method provides a new alternative for multi-resolution surgical simulation with particle-based methods. Since most of the material deformation details are precomputed and stored, our method works well in real-time virtual surgery environment and provides physically valid simulation results.

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